Integer-valued stochastic volatility: structure and Bayesian estimation

Abdelhakim ACKNOUCHE, USTHB and Qassim University

Mots-clés : Parameter driven models, unobserved intensity models, integer-valued stochastic volatility, mixed Poisson log-autoregression, Bayesian MCMC estimation, Griddy Gibbs sampler.

Count time series analysis is nowadays an active area of time series research. Numerous models and methods have been recently introduced in order to account for the main characteristics exhibited by count time series observed in practice such as overdispersion, small values, overfrequency of zeros, locally constant behavior and asymmetric marginal distributions. Following the fundamental classification by Cox (1981), a time series model is called observation-driven or parameter-driven depending to whether the corresponding conditional distribution is specified conditionally on an observable or unobservable process. Since count time series models are almost often related to the Poisson process with a given parametric intensity, Cox's categorization may reduce for integer-valued models to the following dichotomy: observable intensity models versus unobservable intensity models. As is well known, observable intensity models which include in particular integer-valued GARCH (INGARCH) processes are reputed to be easy to estimate, particularly by maximum likelihood-type methods, but their probabilistic structures (e.g., ergodicity, existence of moments) are quite difficult to reveal. In contrast, unobservable intensity models are generally of simple structure and offer a great flexibility in representing serial autocorrelation. However, their estimation by the maximum likelihood method is computationally very demanding, if not infeasible. Moreover, unlike observable intensity models (e.g., INGARCH models) a weak ARMA representation for unobservable intensity models does not exist. Nevertheless, unobserved intensity models as a class of parameter driven models may, in principle, be estimated by filtering-based methods such as Bayesian MCMC and EM-type algorithms.
In this work we propose a class of unobservable intensity models we call integer-valued stochastic volatility (INSV). The corresponding conditional distribution is a Poisson mixture where the logarithm of the intensity follows a drifted Gaussian autoregression. The proposed model allows for a wide range of possible conditional distributions such as Poisson, negative binomial, double Poisson, etc. In the pure Poisson case, the INSV model is the discrete-valued analog of the stochastic volatility (SV) model proposed by Taylor (1982). Our model may be viewed as an alternative to the observation-driven Poisson INGARCH model in which the intensity only depends on the past process. Similar unobserved intensity models have been earlier proposed but they differ from our model by their parametrization. We first study the probability structure of the INSV model such as ergodicity, covariance structure and existence of moments. Then parameter estimation is carried out using the Bayesian Griddy Gibbs sampler in both Poisson and negative binomial cases. In particular, the autoregressive parameters are sampled using conjugate priors while the unobservable intensity and the variance of the mixing process are indirectly sampled using the Griddy Gibbs scheme. The unobserved intensities are sampled element-by-element in the spirit of Jaquier et al. (1994) while model selection is performed using the Deviance Information Criterion. An application to Bayesian intensity forecasting through simulated and real count series is given.

Références

[1] Cox, D.R. (1981). Statistical analysis of time series: some recent developments. *Scandinavian Journal of Statistics* **8**, 93-115.

[2] Jacquier, E., Polson, N.G. and Rossi, P.E. (1994). Bayesian analysis of stochastic volatility models. *Journal of Business and Economic Statistics* **12**, 413-417.

[3] Taylor, S.J. (1982). Financial returns modelled by the product of two stochastic processes- a study of daily sugar prices, 1961-1979. In *Time Series Analysis: Theory and Practice*, Anderson O.D. (ed.). North-Holland, Amsterdam, 203-226.

Abdelhakim ACKNOUCHE, Faculté de Mathématiques, Université des Sciences et de la Technologie Houari Boumediene and Mathematics departement, Qassim University.