Two-stage weighted least squares inference for count time series models

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Consider an integer-valued stochastic process $\{X_t, t \in \mathbb{Z}\}$. Assume a parametric form for the conditional mean :

$$E\left(X_t \mid X_{t-1}, X_{t-2}, \dots\right) = \lambda\left(X_{t-1}, X_{t-2}, ...; \theta_0\right) = \lambda_t\left(\theta_0\right) = \lambda_t, \qquad t \in \mathbb{Z}.$$

For most of the count time series models, in particular the Poisson INteger GARCH (INGARCH), the Negative Binomial INGARCH and the INteger AR (INAR), that conditional mean has a linear form. Estimating θ_0 is obviously of primary importance, in particular for predicting the future values of X_t. The maximum-likelihood estimator (MLE) is often readily computable—except for parameter-driven models like the INAR model—but it requires to specify a conditional distribution (for instance Poisson or Negative Binomial). In practice, the choice of the conditional distribution is an issue. There exists actually no natural choice for the conditional distribution, or even for the condition variance v_t. For example, the choice of the Poisson distribution with intensity λ_t entails $v_t = \lambda_t$, and is thus questionable since it has been empirically observed that numerous count time series exhibit conditional overdispersion (see *e.g.* Christou and Fokianos, 2014). Moreover the choice of a wrong conditional distribution may affect the efficiency, or even the consistency, of the misspecified MLE of θ_0.
In the present work, we focus on the estimation of the parameter θ_0 of the conditional mean, without assuming a specific form for the conditional distribution of the observations (such as for instance the Poisson distribution). In particular, we are interested in estimators that could be consistent even if the conditional variance is misspecified. An example of such misspecification-consistent estimator is the Poisson QMLE (PQMLE). This estimator coincides with the MLE when the conditional distribution of the observations is Poisson $\mathcal{P}(\lambda_t)$, but the PQMLE is consistent and asymptotically normal (CAN) for a much broader class of conditional distributions (see Ahmad and Francq, 2016). However, this estimator is in general inefficient when $v_t \neq \lambda_t$. Motivated by the existence of overdispersed series for which $v_t > \lambda_t$, Aknouche, Bendjeddou and Touche (2018) studied the profile Negative Binomial QMLE (NBQMLE). This estimator is also consistent for estimating θ_0 under very mild regularity conditions, but may be inefficient.
We propose and study alternative weighted least squares estimators (WLSE), which enjoy the same consistency property as the PQMLE and NBQMLE when the conditional distribution is misspecified, but gain in efficiency when v_t is well specified. The asymptotic and finite sample properties of these estimators have been studied. Compare to the above-mentioned QMLEs, the WLSE presents the advantages of 1) being of higher efficiency in some situations; 2) being asymptotically efficient when the conditional distribution belongs to the linear exponential family; 3) having a standard asymptotic normal distribution even when one or several coefficients of the conditional mean are equal to zero; 4) being explicit and requiring no optimisation routine in INARCH models.

Références

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